Numerical Techniques for Topographic Measurement in Water Area

Satoshi Iwakami[1], Masahiko Tamega[1], Shuji Jimbo[2], Masaji Watanabe[3]

[1]Earth Rise Co. Ltd., Okayama, Japan
[2]Graduate School of Natural Science, Okayama University, Okayama, Japan
[3]Specially Appointed Professor, Okayama University, Okayama, Japan
watan-m@okayama-u.ac.jp

Abstract. This study demonstrates numerical techniques for a topographic study of a water area. Data obtained by a GPS unit and an echo sounder were combined, and three dimensional data are generated. A problem to find a surface that contains those data points were reformulated to a problem to find a piecewise linear continuous function defined over a triangular mesh. An iterative method leads to an approximation of a fixed point. Those techniques are illustrated, and numerical examples are introduced.

Key words: Water area topography, Triangular mesh, Fixed point iteration, GPS, Echo sounder

I. INTRODUCTION

Seto Inland Sea is an inland sea between the main island and the Shikoku Island of Japan. Kojima Bay is located in the coast of the Seto Inland Sea along the main island. A part of the Kojima Bay was closed by an embankment to be a reservoir named Kojima Lake. The area of the Kojima Lake is approximately 10 km^2. Inflows from two rivers, Sasagase river and Kurashiki river are primary source of the water in the Kojima Lake. There are six gates set in the bank between the Kojima Lake and the Kojima Bay. The water level of the Kojima Lake is maintained by the discharge through the gates into the Kojima Bay. The topography of such a reservoir is subject to change due to erosion and sedimentation, and updated topographic data should be provided on a regular basis.

Topographic surveys of the Kojima Lake were conducted on September 28th and October 4th in 2019. Positional data were recorded by a real time kinematic GPS (RTK-GPS) unit with a virtual reference station (VRS) system, while depth data were recorded by an echo sounder mounted on a vessel (Figure 1).

(a) A GPS antenna attached to one end of a pole.

(b) An oscillator of an echo sounder was attached to the other end of the pole beneath the water surface.

FIGURE 1. RTK-GPS and echo sounder.

Positional data on the reference ellipsoid expressed in terms of longitude and latitude were transformed to rectangular coordinates with the Gauss-Krüger Projection. Those data were combined with the depth data to be three dimensional topographic data.

Those data obtained from the topographic measurement were introduced into a topographic analysis. A triangular mesh in the plane was set. The problem to find a surface that contains the measurement data was reformulated into a fixed point problem of a map on values on nodes in a triangular mesh. Numerical techniques are illustrated, and numerical results are introduced.

II. Numerical Techniques for Topographic Analysis

A. RTK-GPS Data and Depth Data

Topographic data of the Kojima Lake were obtained in the measurement conducted on September 28th and October 4th. The positional data were obtained by the RTK-GPS. Ellipsoidal coordinates given in terms of the longitude and the latitude were transformed to rectangular coordinates with the Gauss-Krüger Projection. Figure 2 shows the RTK-GPS tracks in the plane.

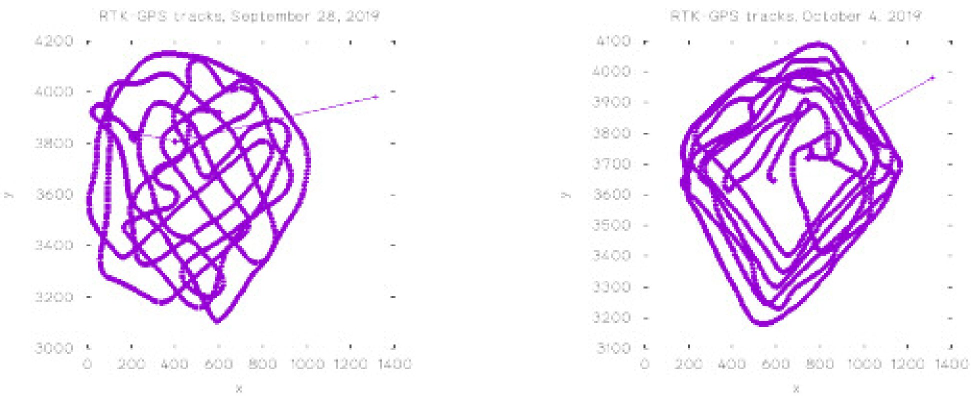
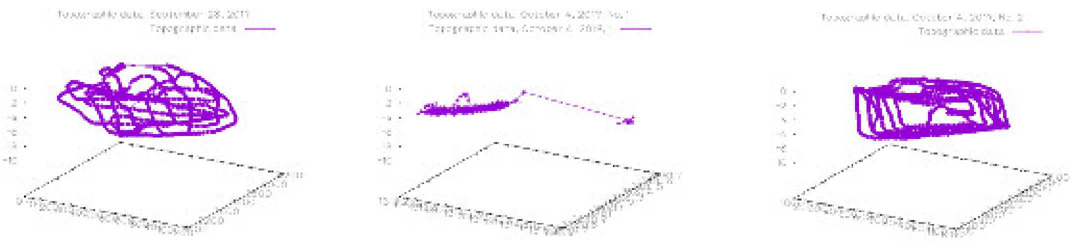


FIGURE 2. RTK-GPS tracks

The RTK-GPS data were combined with the depth data recorded by the echo sounder to be three dimensional topographic data were generated (Figure 3).



(a) Septermber 28th , 2019 (b) October 4th, 2019, No. 1 (c) October 4th, 2019, No. 2

FIGURE 3. Three dimensional topographic data.

B. Formulation of Map of Node Date and Fixed Point Iteration

Let m be the number of elements and n be the number of nodes in a triangular mesh in the plane. Figure 4 shows a triangular mesh over a region that contains all the significant GPS tracks. Consider a topography expressed by a graph of a continuous function of x and y, $z = f(x,y)$, whose domain is the region over the triangular mesh. Consider the vector space of piece wise linear continuous functions that are linear in each of elements. Suppose that the element k contains (x,y) components of the l data points, (x_1, y_1, z_1), (x_2, y_2, z_2), … , (x_l, y_l, z_l). Suppose that those data points include

$$(x_{k,1}, y_{k,1}, z_{k,1}),\quad (x_{k,2}, y_{k,2}, z_{k,2}),\quad (x_{k,3}, y_{k,3}, z_{k,3})$$

where $(x_{k,1}, y_{k,1})$, $(x_{k,2}, y_{k,2})$, $(x_{k,3}, y_{k,3})$ are the vertices of the element k.

The values of coefficients a, b, and c of the linear function $z = ax+by+c$ for the element k were those that minimize the square sum

$$\sum_{j=1}^{l}[z_i - (ax_i + by_i + c)]^2.$$

Once those coefficients were determined for all the elements, the value of the z components at each node was determined as the area weighted average of all the values of linear functions over all the elements surrounding it. Thus a map on the n dimensional space was defined.

It took 3244 fixed point iterations till the residual error reduced to 0.001. Figures 5 and 6 show numerical results.

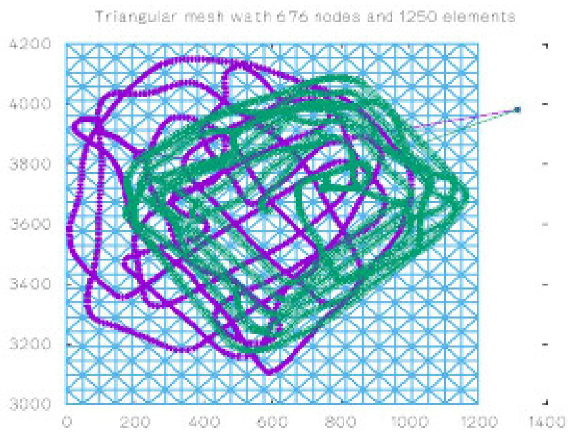


FIGURE 4. Triangular mesh

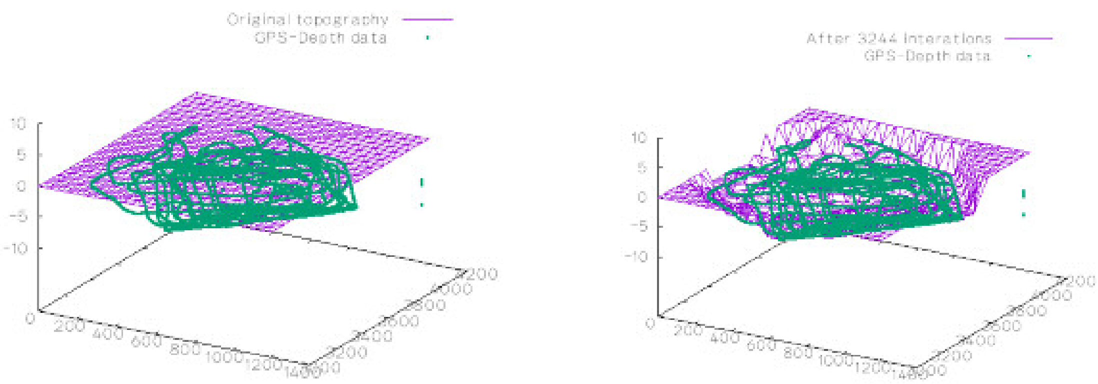


FIGURE 5. Original surface and surface after 3244 fixed point iterations.

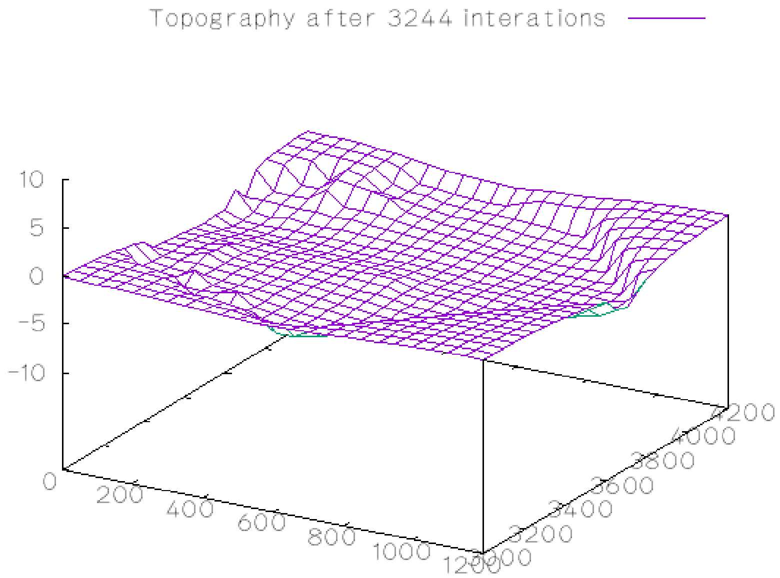


FIGURE 6. Surface after 3244 fixed point iterations.

III. DISCUSSION

REFERECES

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